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Water Hammer in pipes

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# WATER HAMMER IN PIPES

BY

MELVIN LORENIUS ENGER

## THESIS

FOR

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IN

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This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Theoretical and Applied Mechanics, by

MELVIN LORENIUS ENGER

entitled WATER HAMMER IN PIPES

is hereby approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Ira O Baker

Head of Department of Civil Engineering

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# Water Hammer in Pipes

It has long been known that the sudden closure of a valve at the end of a long pipe, produces a pressure liable to burst the pipe. In the design of pipes and valves a knowledge of the possible pressure is necessary. A number of experiments have been made and formulas developed, so that the maximum pressure under given conditions is readily found. However, the phenomenon is far from being well understood.

This thesis was undertaken to explain:—

(1) The decrease in pressure near the open end of the pipe.

(2) Why the time between successive impulses becomes less.

(3) Why the time obtained by dividing the average pressure in the pipe by the velocity of the water before closing the valve, is not equal to the time it takes wave to run the length of the pipe.

And to investigate,

(4) The effect of a leak on the pressure in the pipe beyond the point of leak, and the experimental determination of the place of leak by means

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of the pressure diagram.

(5) The effect of the time of closure on the pressure diagram

(6) The effect of placing in the line a piece of pipe of larger diameter.

## Existing Literature

The literature on the subject of water hammer is surprisingly small. Much of it is purely theoretical and based on incorrect assumptions, while the experimental work, with one exception, has been on short lengths of pipe.

### Joseph B. Rider

In the Proceedings of the American Water-Works Association of April, 1891, Joseph B. Rider gives an account of his experiments and theoretic investigation of the subject. His experiments were mainly on 18 feet of pipe using an ordinary gage to measure the pressure. He considered the water in the pipe as an inelastic column whose total kinetic energy was given up during the time of closing the valve. He then applies his principles to the problem of 22000 feet of 12 inch pipe, water flowing with a velocity of .424 feet per second, and stoppage occurring in two seconds. The resultant pressure he figures as 3 794.65 pounds per square inch. The pressure would in reality be only

about 20 pounds per square inch.

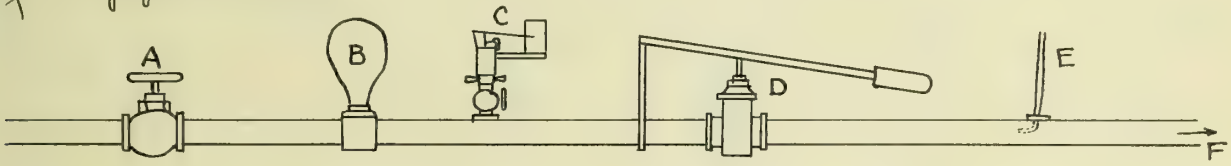
## Professor Church.

In 1890, Prof. Church published an account of his experiments on water hammer, in the Journal of the Franklin Institute. He came to the conclusion that the pressure developed was a direct function of the time and manner of closing the valve. We shall see that the time and manner of closing the valve has no effect whatever as long as the closure is made in less time than it takes pressure waves to run from the valve to the open end and back.

## Professor Carpenter.

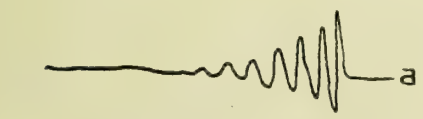
More elaborate experiments were made at Sibley College under the direction of Professor Carpenter. The results were published in the Transactions of the American Society of Mechanical Engineers, 1894 Vol. 15, p. 150, and in the Engineering Record, Vol 30, 1894 under the heading "Some Experiments on the Effect of Water Hammer." As the experiments of Carpenter are the ones most often quoted, they merit

some consideration here. The experiments were made on one and a half ($1\frac{1}{2}$) inch pipe, the water having velocities up to 8.6 feet per second The following figure shows the arrangement of apparatus.

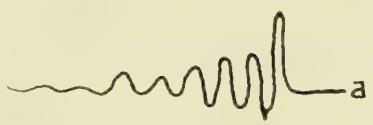



The velocity of the water was regulated by the valve A. An air chamber B was used in some of the experiments. The changes in pressure were measured by a Crosby indicator C, and at E was placed a pitot tube to measure the velocity of the water.

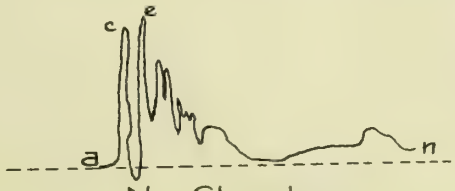
Experiments were made with an air chamber, a water chamber and with no chamber. The following diagrams are representative.




Air Chamber




Water Chamber




No Chamber

The beginnings of the diagrams are marked with a letter a. The smoothest diagram is the one with an air chamber. By increasing the volume of air, the greatest pressure was diminished, but not in proportion to the volume of the air chamber. The lower diagram shows a rapid rise to a point c, from which it falls below atmospheric pressure, and then again rose sharply to e, which was usually higher than c. Beyond this point a very marked trembling of the indicator pencil is seen. The difference in the heights c and e is probably due to the different static heads on the two sides of valve A, the sharp depression being a record of the passage of the wave through the valve. The trembling of the indicator was undoubtedly caused by air in the pipe.

The following is a table of Carpenter's results on a line of pipe with no air or water chamber

| Velocity of Water in Feet pr. Second | Excess Pressure Pounds pr. sq. in. |
|---|---|
| 2.91 | 63 |
| 3.35 | 90 |
| 4.20 | 113 |
| 5.05 | 143 |
| 6.02 | 196 |
| 7.07 | 231 |
| 8.60 | 254 |

The pressures are all smaller than those which later experiments have determined. This is because for the short length of pipe used it was not possible to close valve in the time it takes wave to run to the open end and back.

## Elmer Church Smith, 1899

Mr. Elmer Church Smith '99 presented a thesis on "Water Ram in Pipes" Up to about this time no attempt had been made to determine anything else than the maximum pressure, and some way of mitigating it. Mr. Smiths experiments were on 205 feet of two inch steel pipe. He arrived at the following conclusions:

1. The pressure varies as the time of closing the valve
2. The pressure varies directly as the velocity of the water flowing in the pipes
3. The pressure varies nearly as the square root of the distance from the valve.
4. The pressure is independent of the diameter and length of the pipe.

Conclusion 2 has been proved correct, 4 is right with regard to length, 1 and 3 are surely erroneous

## B. W. Seymour, 1903

Mr. Budd Willard Seymour, '03 presented a thesis at the University of Illinois, on the subject of "Water Hammer in Pipes". Mr. Seymour experimented on 180 and 300 feet of two inch steel pipe. He drew the following conclusions from his experiments:

1. The pressure created by the sudden closure of the valve is nearly proportional to the velocity of the water flowing in the pipe.
2. The pressure is greatest at the valve, and zero at the free end. It decreases, not as the distance from the valve increases, but nearly as the square root of the distance
3. The pressure is not one shock, but several successive shocks at decreasing intervals, the time of these intervals increasing as the length of pipe, and as the velocity of the water.
4. The pressure does not occur simultaneously at all points along the pipe, but is first felt at the valve, and this impulse travels along the pipe with a velocity of 4600 feet per second.
5. The time that the compression lasts at any

point is the time it takes the impulse to travel from that point to the free end, and to return to the point in question.

6. The computed time that the water continues to flow into the pipe after the valve is closed is not the same as the time it takes the impulse to travel from the valve to the free end.

It will be seen that Mr. Seymour in conclusion 2 makes the same mistake as Mr. Smith, in stating that the pressure varies as the square root of the distance from the valve. They mistook for a general law what was only the special case of a short length of pipe.

## Torris Eide, 1904.

Mr. Torris Eide '04 presented a thesis at the University of Illinois on the subject of "Water Hammer in Pipes." Mr. Eide used 300 feet of two inch steel pipe, putting in a quick-closing valve 180 feet from the valve at the end. He made experiments closing the two valves simultaneously, closing second valve so as to confine impulse on return from the stand-pipe.

These experiments were not very successful. He also made experiments with a plunger, which was struck by a pendulum weighing 94 pounds, dropped from various heights.

Mr. Eide came to the conclusion that the pressure at various points of the pipe is not a function of the distance from the valve, but that for long lines of pipe the pressure would remain constant until near the entrance, when it decreases rapidly.

## Prof. N. Joukowsky

In 1897 the engineers of the waterworks system of Moscow ordered an investigation of the subject of water hammer. Professor N. Joukowsky was given charge of the experiments.

He used pipes of 2, 4 and 6 inches diameter, with lengths of 2494, 1050 and 1066 feet. He made numerous experiments and deduced formulae which seem to be in accord with the facts. The monograph which Joukowsky presented the St. Petersburg Academy of Sciences has been translated into German (Stoss in Wasserleitungsröhren, St. Petersburg 1900)

Joukowsky concludes that:-

(1) The hydraulic shock moves along the pipe with a constant velocity; the amount of which, as far as noticed, does not depend on the amount of shock. This velocity does depend upon the material of the pipe and the ratio of the thickness of the wall to the diameter. Since in ordinary cast-iron pipe the above ratio diminishes a little as the diameter increases, the velocity of the shock for the large diameters will be somewhat less than for small pipe. For small pipe (from 2 to 6 inches) the velocity is about 4200 feet, for large pipe (24 inches), about 3290 feet pr. second.

(2) The hydraulic shock is transmitted with equal pressure through the pipe. The pressure depends on the velocity of flow of the water, and the velocity of the transmission of the pressure wave through the pipe. For ordinary cast-iron we obtain about 4 atmospheres per square inch for every foot of lost velocity, in pipes of small sizes (from 2 to 6 inches), and for 24 inch cast iron pipe about 3 atmospheres.

(3) The appearance of periodic vibrations of the

pressure wave is completely explained by the reflection of the wave from the end, and the beginning of the pipe.

(4) The flow of water in the pipe after a partial closure has no noticeable effect on the pressure. The pressure depends only on the lost velocity. In the case of a pressure wave moving back in a pipe from which water is flowing, the reflection from the beginning of the stream takes place the same as if reflected from a standpipe or main under constant pressure.

A dangerous increase of pressure occurs upon the transition of the pressure wave from a pipe of large diameter to a smaller one. The pressure of the shock wave is doubled on reaching a dead end. This doubling may take place several times, so that under favorable circumstances a very great pressure may be obtained.

(6) The simplest means of protecting pipe from the shock is by the slow closing of all valves

To do this the time of closing must be pro-

portional to the length of pipe. Air Chambers of proper dimensions, placed at the valve, lessen the shock very considerably, but keeping air in the chambers is a very difficult matter. A relief valve allows only such pressures to pass as are equal to the strength of its spring.

(7) From the pressure diagram of a line of pipe, the place of accumulation of air in the pipe, and its volume may be determined. The diagram will also serve as a means of locating the place of a leak in the pipe, and give general information as to the condition of the pipe.

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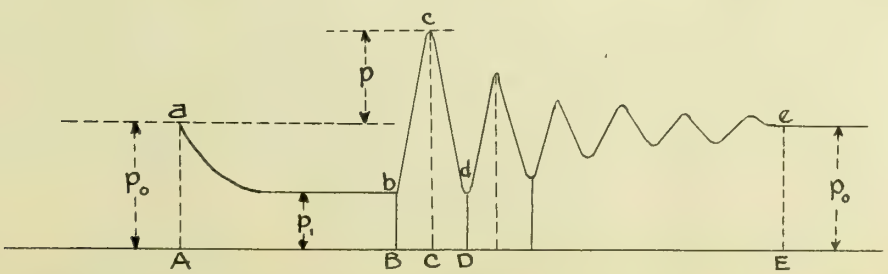
# Theoretic Determination of Maximum Pressure

The following is from "A Treatise on Hydraulics" by

Mansfield Merriman

"When a valve in a pipe is closed while the water is flowing, the velocity is retarded and thus a dynamic pressure is produced. When the valve is closed quickly, this dynamic pressure may be much greater than that due to the static head, and it is then called "water hammer" or "water ram." Pipes have often been known to burst under this cause, and hence the determination of the maximum dynamic pressure of water hammer is a matter of importance. The figure illustrates the phenomena of water hammer for the closing of a valve at the end of a pipe where the water issues through a nozzle. At the entrance there is supposed



to be a gage which registers the static unit-pressure $p_1$, while the flow is in progress, and the static unit-pressure $p_0$ when there is no flow. The abscissas represent time, and at B the valve begins to close. After a short interval of time BC the gage registers the unit-pressure Cc; after another short interval the unit-pressure has decreased to Dd, and a series of oscillations follows until finally the disturbance ceases. A diagram of this kind may be autographically drawn by a suitable mechanism connected with the pressure gage, and such were made in the experiments conducted by Carpenter, as also in those of Fletcher.

Let P represent the excess of maximum unit-pressure over the static unit-pressure when there is no flow; that is, the difference of the ordinates Cc and Ec. This is the excess unit-pressure due to the water hammer and it is required to determine an expression for its value. It is first to be noted that the actual dynamic unit-pressure produced by the retardation of the velocity is the difference of the ordinates Cc and Bb and this difference is

$$p + p_0 - p_1$$

The dynamic pressure on the area a of the cross-section of the pipe is then

$$(p + p_0 - p_1)a,$$

and for brevity this may be represented by P. If this pressure be regarded as varying uniformly from 0 to P during the time t in which the valve closes, its mean value is $\frac{1}{2}P$ and its total impulse during this time is $\frac{1}{2}Pt$. If l be the length of the pipe, w the weight of a cubic unit of water, and v the velocity during the flow, the total weight of water in the pipe is wal and its impulse is

$$wal\frac{v}{g}$$

Equating these expressions of impulse there is found

$$P = \frac{2wal}{gt}$$

and replacing P by its value, there results

$$p = \frac{2wl}{gt}v + p_1 - p_0 \quad \text{.....(1)}$$

as the excess dynamic unit-pressure due to closing the valve in the time t. This formula, having been deduced without considering the fact that time is required for the transmission of stress through water, cannot be regarded as applicable to all cases.

It has been shown that the velocity with which any disturbance is propagated through water is about 4670 feet per second, and this velocity may be represented by $u$. Now let the pipe of length $l$ have an open valve at the end, and let the water be flowing through every section with a velocity $v$. Then the time $\frac{l}{u}$ must elapse after valve begins to close before the velocity begins to be checked at the upper end of the pipe, and the further time $\frac{l}{u}$ must elapse before the pressure due to this retardation can be transmitted back to the valve. The total time $\frac{2l}{u}$ is then required before the gage at the valve can indicate the pressure due to retardations of the velocity in the length $l$. Hence, if the time in which the valve closes be equal to, or less than $\frac{2l}{u}$, the time $t$ in the above formula is to be replaced by $\frac{2l}{u}$ and thus

$$p = \frac{wu}{g}v + p_1 - p_0 \quad \text{.....(2)}$$

is the maximum excess dynamic unit-pressure that can occur in the pipe. This depends upon the velocity of the water and upon the initial and final static pressures.

The subject of water hammer is one of

the most difficult in hydro-mechanics, and the above investigation cannot be regarded as final. Formula 1 is probably correct only for a certain law of valve closing. Formula 2, however, is certainly correct, for it may be proved by other methods, one of which is as follows: When water is in motion the kinetic energy in a length $\delta l$ at the gage is

$$w a \delta l \frac{v^2}{2g};$$

when it is brought to rest under a unit stress $s$, its stress energy is

$$a \delta l \frac{s^2}{2E}$$

if $E$ be the modulus of elasticity of the water. Equating these expressions, and substituting $p + p_1 - p_0$ for $s$ there results for excess dynamic unit-pressure

$$p = \left(\frac{Ew}{g}\right)^{\frac{1}{2}} v + p_1 - p_0$$

and this reduces to formula 2 if $E$ be replaced by $\frac{wu^2}{g}$, which is its value according to a previous proof

When $v$ is in feet per second, and $p_0$, $p_1$ and $p$ are in pounds per square inch, these formulas become

$$p = 0.027\left(\frac{l}{t}\right) v + p_1 - p_0 \qquad p = 63 v + p_1 - p_0$$

the first of which is used when t is greater than 0.000428 l and the second when t is equal to or less than 0.000428 l, l being in feet"

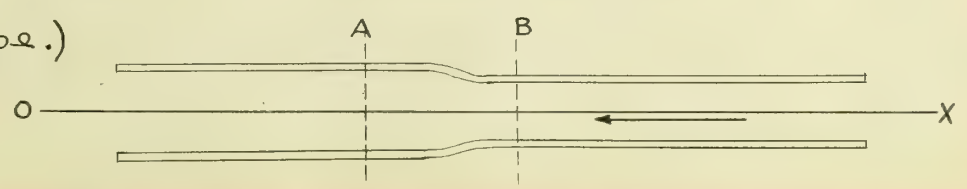
It should be noticed in the foregoing discussion that Prof. Merriman takes no account of the elasticity of the walls of the pipe.

The following is a somewhat modified form of the discussion of

## Prof. N. Joukowsky

After the closing of the valve a shock will be sent through the water in the pipe with a velocity $\lambda$.

The figure is a section of the pipe, and shows a part expanded by the pressure of the water hammer. Assume that the sections A and B are so placed that at a given time the velocity of the water through A is zero, and through B is $v_0$. The pressures at A and B are then the maximum p, and the normal $p_0$, respectively (For simplicity we will assume the pressure the same at all points in the pipe.)



The volume of the water which flows through the section B is equal to

$$\pi R^2 v_0 dt \quad \text{----------(1)}$$

and this will find place between A and B; while during the time dt, the point at which the deformation of the pipe, and the change of density of the water, commences, has moved forward a distance $\lambda dt$ The increased quantity of water due to this cause will be

$$\pi (R^2 - R_0^2) \lambda dt + \pi R_0^2 \left(\frac{\rho - \rho_0}{\rho_0}\right) \lambda dt \quad \text{---------(2)}$$

$\rho_0$ and $\rho$ being the densities before and after the increase in pressure, and $R_0$ and $R$ the radii. The ratio

$$\frac{\rho - \rho_0}{\rho_0}$$

expresses the change in the density of the water.

Equating 1 and 2.

$$\pi R^2 v_0 dt = \pi (R^2 - R_0^2) \lambda dt + \pi R_0^2 \left(\frac{\rho - \rho_0}{\rho_0}\right) \lambda dt \quad \text{-------(3)}$$

Denoting the elasticity of water by k we may write,

$$p - p_0 = \left(1 - \frac{\rho_0}{\rho}\right) k$$

or, since $\rho_0$ and $\rho$ are nearly equal

$$p - p_0 = \left(\frac{\rho - \rho_0}{\rho_0}\right) k \quad \text{----------(4)}$$

Considering a length $dx$ of the pipe we have from the formula $2\ell tS = Pd\ell$

$$2eE\,dx\,\frac{R-R_0}{R_0} = 2R\,dx\,(p-p_0)$$

or

$$p-p_0 = \frac{eE}{R_0^2}(R-R_0) \qquad \text{----------(5)}$$

$E$ being the modulus of elasticity of the iron composing the pipe, and $e$ the thickness of the walls of the pipe.

Substituting equations 4 and 5 in 3 we obtain the following expression

$$v_0 = (p-p_0)\left(\frac{2R_0}{eE}\lambda + \frac{1}{k}\lambda\right)$$

From this formula the increase in pressure may be found. Denoting it by $P$, we have

$$P = \frac{v_0}{\lambda\left(\frac{2R_0}{eE}+\frac{1}{k}\right)} = \frac{v_0}{\frac{\lambda}{\rho_0}\left(\frac{2R_0\rho_0}{Ee}+\frac{\rho_0}{k}\right)} \qquad \text{----------(6)}$$

The expression for the velocity $\lambda$ in a compressible fluid in an elastic pipe is, although no proof will be attempted here,

$$\lambda = \frac{1}{\sqrt{\frac{\rho_0}{k}+\frac{2R_0\rho_0}{Ee}}}$$

Substituting in 6, and remembering that $\rho_0 = \frac{\gamma}{g}$

$$P = \frac{v_0\lambda^2\rho_0}{\lambda} = \lambda\rho_0 v_0 = \frac{\lambda\gamma}{g}v_0 \qquad \text{-------(7)}$$

From which it is seen that the increase in pressure is proportional to the velocity lost by the water, and to the velocity $\lambda$ of the pressure wave.

$y$ and $g$ represent the weight per cubic unit of the water and the acceleration of gravity.

This formula takes into account the elasticity of the metal in the pipe, and agrees very well with the results of experiments.

A much neater derivation of an expression, given below, is due to

## Professor A. N. Talbot

Suddenly arresting the flow of water in a pipe causes the water near the valve to be compressed a certain amount, while at the free end there is no compression. Neglecting the distention of the pipe at present;. let $dx$ be the length of a column of water whose cross-section is 1 square foot. Let $P$ equal the pressure at the valve on 1 square inch, generated by the sudden stoppage of the water; then the pressure on 1 square foot equals $144P$. The weight of a column of water, 1 square foot cross-section and $dx$ long, is $wdx$; $w$ being the weight of a cubic unit of water. Flowing with a velocity $V$, the energy in this water is equal to $\frac{1}{2}MV^2$. This becomes

$$\text{Energy} = \frac{1}{2}\frac{wdx}{g}V^2$$

Let $\lambda$ equal the amount this dx length is compressed at the valve. Since $\lambda$ is proportional to the unit stress

$$\lambda = \frac{Sl}{E}$$

where $S = P$, $l$ = length of column = dx, and $E$ = the modulus of elasticity of water = 300 000 pounds per square inch. If dx is expressed in feet, the product, Energy, will be in foot pounds.

The work done is equal to the average presaure multiplied by the distance moved through. Since the pressure increases from 0 to 144 P, the average pressure equals $\frac{144P}{2}$. The work done is equal to

$$\frac{144P}{2}\lambda = \frac{144P}{2} \times \frac{Pdx}{E}$$

The work done is also equal to the energy in the water, so equating, we get

$$\frac{144P}{2} \cdot \frac{Pdx}{E} = \frac{1}{2}\frac{wdx}{g}V^2$$

from which

$$P = \sqrt{\frac{wE}{144g}} \cdot V$$

Substituting the numerical values for w, E and g, we have

$$P = 63.6\,V$$

This is the value for the pressure at the valve, generated by sudden stoppage of the water, when the elasticity of the metal is not taken into account

Considering the elasticity of the metal in the pipe, the deformation $\lambda_2$ of the pipe is

$$\frac{\pi d S}{E'}$$

but,

$$Pd = 2tS \quad \text{or} \quad S = \frac{Pd}{2t}$$

where $P$ = pressure per unit area, $d$ = diameter of the pipe, $t$ = the thickness of the pipe, and $S$ = tensile stress of the metal in the pipe in pounds per square inch, $E'$ = the modulus of elasticity of the metal = $30\,000\,000^{\#}/\square''$

Then

$$\lambda_2 = \frac{\pi d^2 P}{2t E'}$$

The total lateral pressure $F_2$ in the pipe, for a length $dx$

$$F_2 = \frac{Pd}{2} dx$$

The work done in expanding the pipe is

$$\frac{1}{2} F_2 \lambda_2 = \frac{1}{2} \frac{Pd}{2} \cdot dx \cdot \frac{\pi d^2 P}{2t E'},$$

which becomes

$$\frac{1}{8} \frac{\pi P^2 d^3}{t E'} \cdot dx$$

The work done in compressing the water is

$$\frac{1}{2} F_1 \lambda_1 = \frac{1}{2} \frac{\pi d^2}{4} \times \frac{P^2}{E} dx$$

which becomes

$$\frac{1}{8} \frac{\pi P^2 d^2}{E} dx$$

The energy of the water is

$$\frac{1}{2} \frac{w}{g} \cdot \frac{\pi d^2}{4} \cdot dx \cdot V^2$$

Equating energy to work done, we have

$$\frac{1}{8} \frac{w \pi d^2}{g} dx V^2 = \frac{1}{8} \frac{\pi P^2 d^3}{t E'} dx + \frac{1}{8} \frac{\pi d^2 P^2}{E} \cdot dx$$

reducing

$$P^2 = \frac{1}{1 + \frac{E d}{E' t}} \cdot \frac{w E V^2}{g}$$

or

$$P = \frac{1}{\sqrt{1 + \frac{E d}{E' t}}} \cdot \sqrt{\frac{w E}{g}} \cdot V$$

giving the pressure per unit area expressed in homogeneous units.

Expressing P in pounds per square inch, and V in feet per second, the above equations reduces to

$$P = \frac{1}{\sqrt{1 + \frac{E d}{E' t}}} \sqrt{\frac{w E}{144 g}} \cdot V$$

The form of this formula is the same as the one where the elasticity of the pipe was neglected, the constant $\frac{1}{\sqrt{1 + \frac{E d}{E' t}}}$, depending upon the size of the pipe and its thickness, being the only difference

Using $d = 2.06$ inches, and $t = 0.154$ inches, the values for the 2 inch pipe used in these experiments, the formula becomes

$$P = 59.7V$$

## Comparison of the Formulas of Joukowsky and Talbot.

It may easily be shown that the formula of Joukowsky,

$$P = \frac{v\lambda\gamma}{g}$$

is the same as the one derived by Talbot in an entirely different manner.

Substituting for $\lambda$ its value

$$\frac{1}{\sqrt{\frac{\rho_0}{k} + \frac{2R_0\rho_0}{Ee}}}$$

we obtain

$$P = \frac{1}{\sqrt{\frac{\rho_0}{k} + \frac{2R_0\rho_0}{Ee}}} \cdot \frac{\gamma}{g} \cdot v$$

but since $\frac{\gamma}{g} = \rho_0$

$$P = \frac{1}{\sqrt{\frac{\gamma}{gk} + \frac{2R_0\gamma}{Eeg}}} \cdot \frac{\gamma}{g} \cdot v$$

$$= \frac{\sqrt{k}}{\sqrt{1 + \frac{k \cdot 2R_0}{Ee}}} \sqrt{\frac{\gamma}{g}}$$

or finally

$$P = \frac{1}{\sqrt{1 + \frac{k \cdot 2R_0}{Ee}}} \sqrt{\frac{ky}{g}} \cdot v$$

which except for notation is exactly the same as

$$P = \frac{1}{\sqrt{1 + \frac{Ed}{E't}}} \sqrt{\frac{wE}{g}} \cdot V$$

which was denied by Professor Talbot.

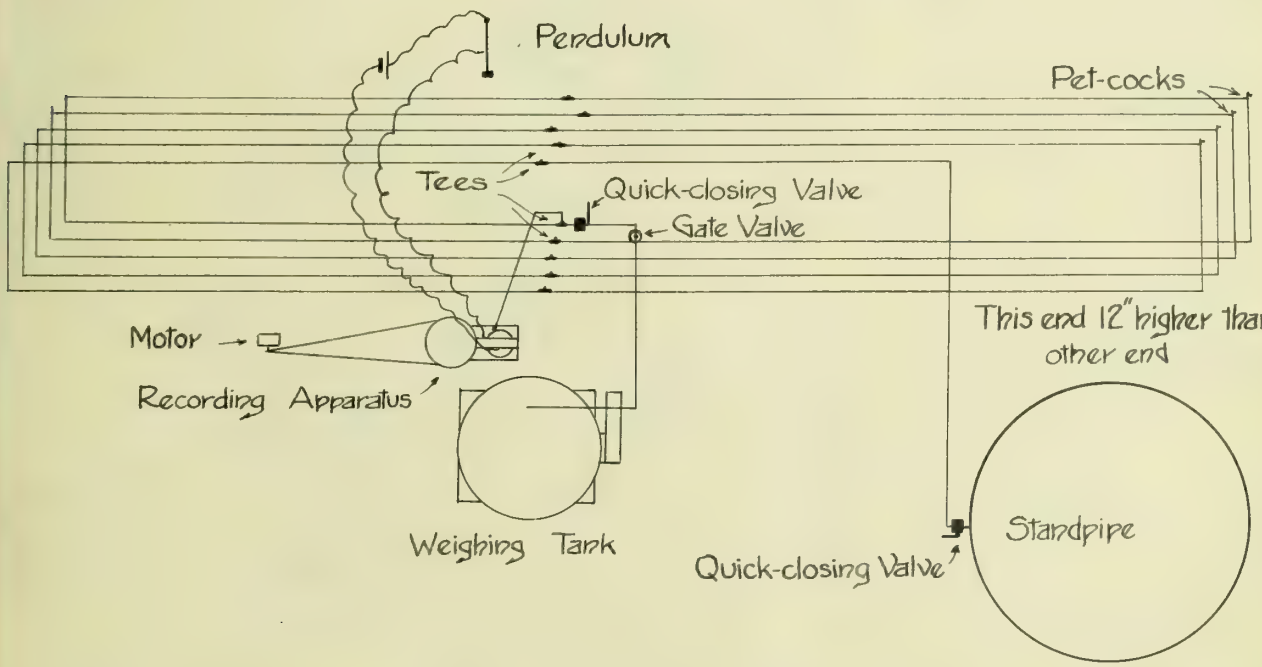
The notations used are:

| Joukowsky | | Talbot. | |
|---|---|---|---|
| $k$ | = | $E$ | = Elasticity of water |
| $E$ | = | $E'$ | = " " metal composing pipe |
| $2R_0$ | = | $d$ | = Diameter of pipe |
| $e$ | = | $t$ | = Thickness of pipe |
| $\gamma$ | = | $w$ | = Weight of a unit volume of water |
| $g$ | = | $g$ | = Acceleration of gravity |
| $v$ | = | $V$ | = Velocity of water in pipe before closure |

## Apparatus

The experiments were made on 727 feet of 2-inch, steel pipe. For convenience the pipe was laid in rectangles about seventy feet by two feet, piled on top of each other. 3/4 inch tees were placed in each long length of pipe, and near together. This made an average distance of about 72 feet between each tee. This arrangement made it easy to make a connection for the indicator to these points.

The following figure shows the arrangement of the pipe and apparatus.



The velocity of the water in the pipe was determined by measuring the time required for 1020 pounds to flow into the measuring tank. A curve was constructed from which the velocity could be found with little labor when the time was known.

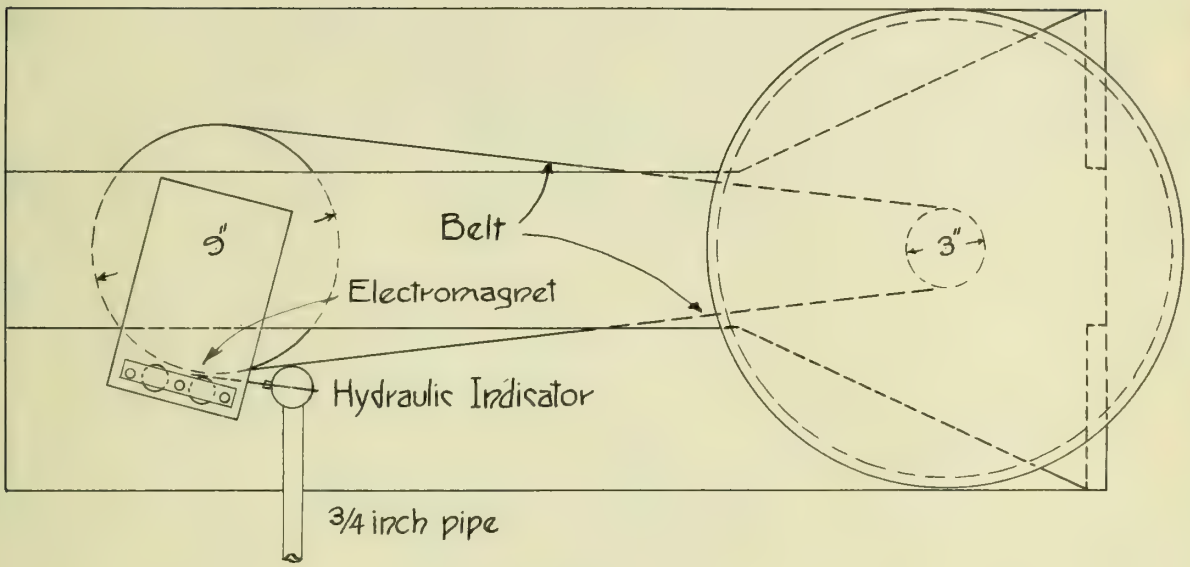
The pipe was laid about one foot higher at one end than the other, and pet cocks were placed at the highest points so that air could be removed.

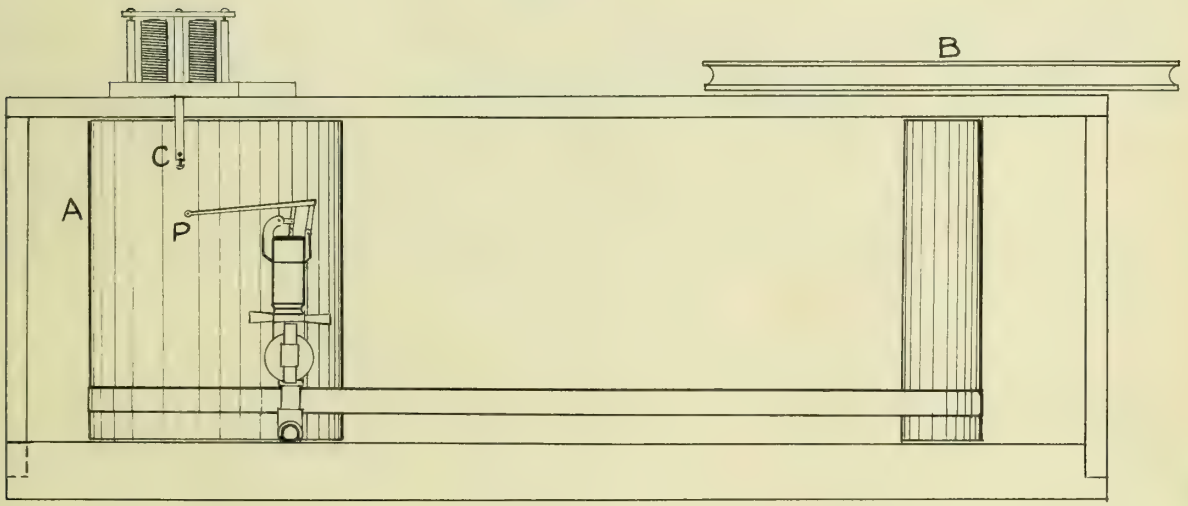
A Crosby indicator was used in measuring the pressure, a 200 pound and a 480 pound spring being used. Some experiments were also made with lighter springs (40 and 60 pound)

The sketch on page 31 shows the plan of the recording device. The pressures are recorded by the indicator pencil P, on a sheet of paper wrapped around the 9-inch drum A, and fastened with thumb-tacks. The drum is driven by a tape belt. The belt is in turn driven by a cotton rope-drive, running over the 18-inch pulley B to an electric motor, which runs at a nearly constant speed

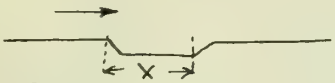
The speed of closing the valve is recorded by a pencil C marking on the sheet of paper

# Recording Apparatus



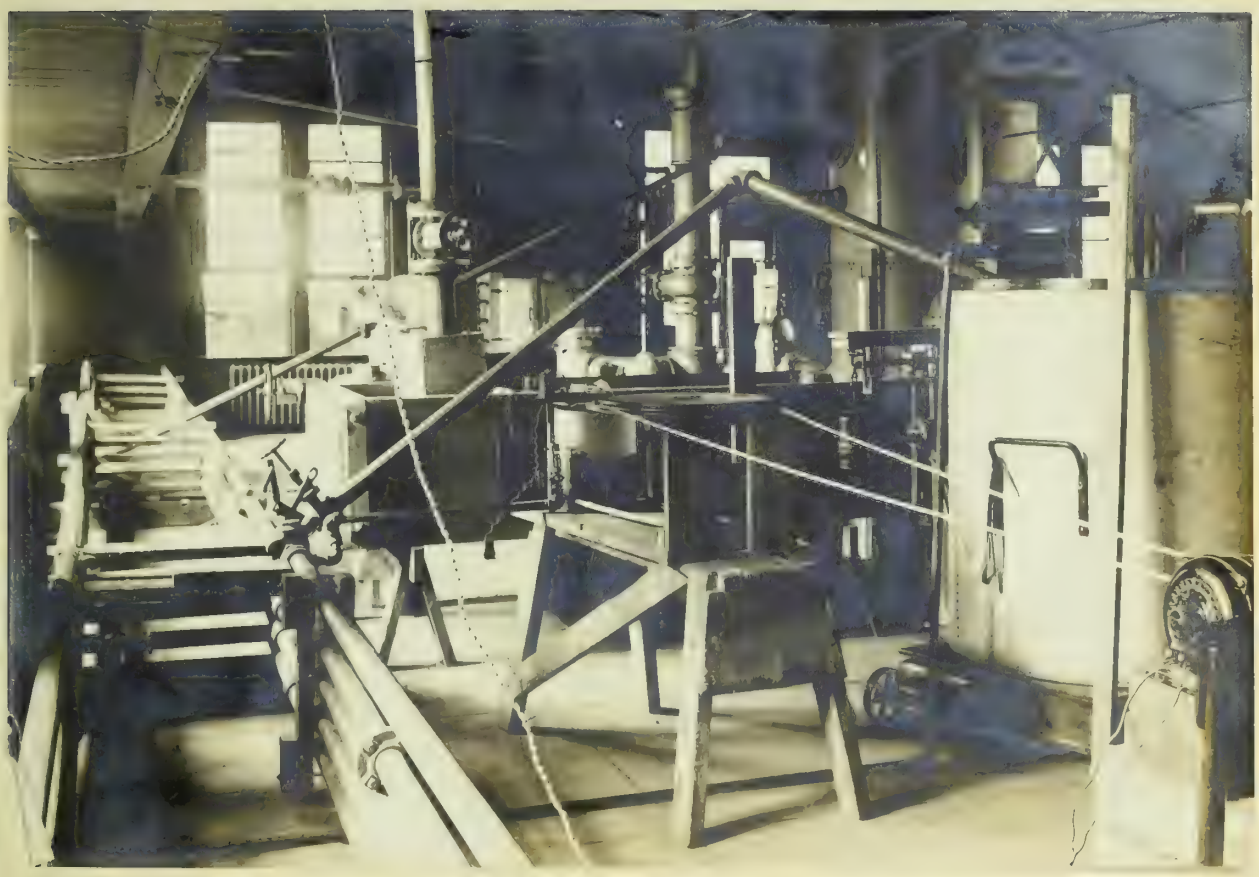



around the drum. This pencil is attached to an electro-magnet so that the electric connection made when the valve begins to close, causes the pencil to move down; when the connection is broken at the end of closure the armature is pushed back by a spring, thus raising the pencil. A line like figure,




is made and the distance x is a measure of the time in which the valve is closed.

This pencil was also actuated by a pendulum which sent out an electric pulsation every 0.7 second. This was necessary because a number of speeds were used, and because motor was very small ($\frac{1}{20}$ H.P.) and small changes of load made considerable changes of speed.



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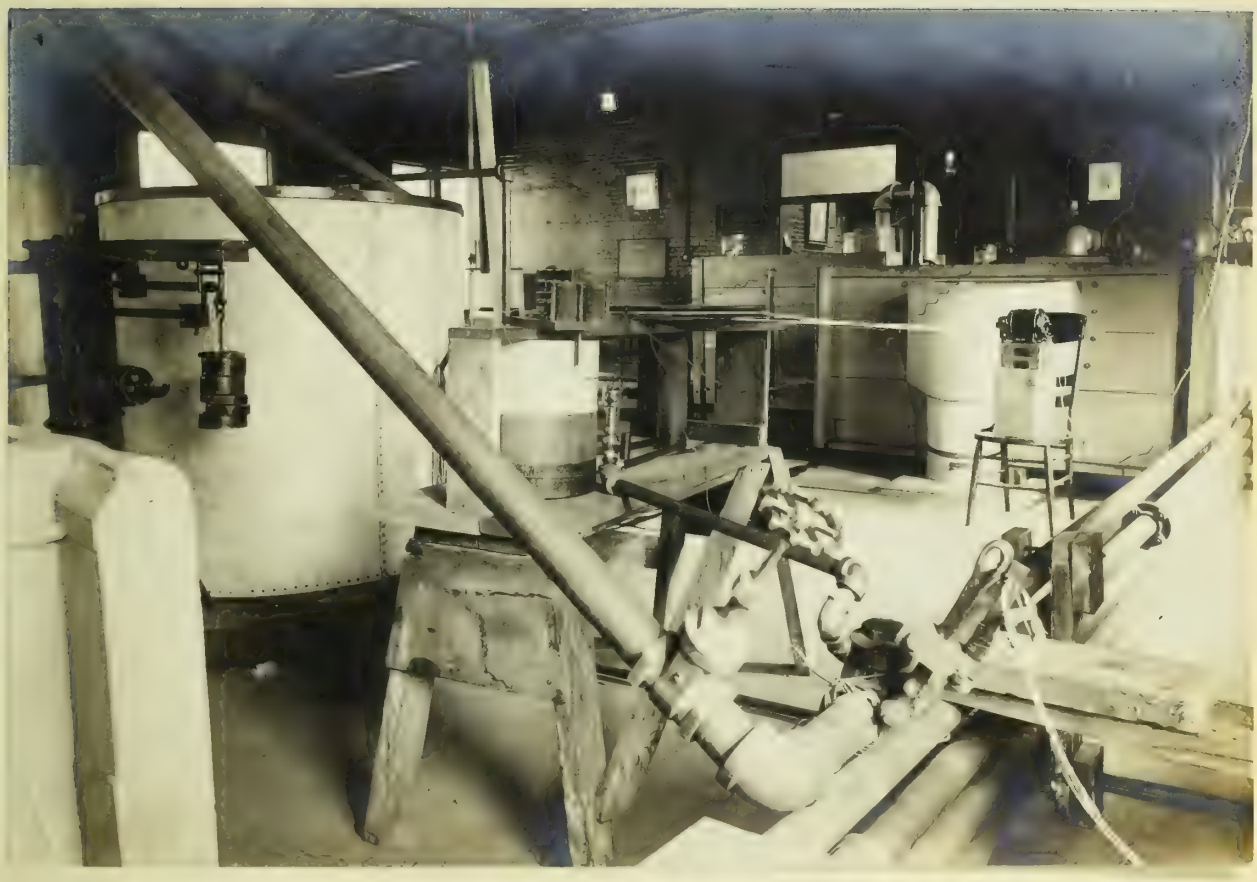

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# Relation of Pressure to Extinguished Velocity

About seventy experiments were made under various conditions of velocity and head, and they all showed a close agreement with the formula derived by Professor Talbot. A few of the results are given below in tabular form.

| No. | P=59.7V Theo. Pressure | Actual Pressure |
|---|---|---|
| 1 | 298 | 290 |
| 2 | 155 | 160 |
| 3 | 143 | 140 |
| 4 | 60 | 63 |
| 5 | 53 | 49 |
| 6 | 33 | 33 |
| 7 | 14 | 17 |
| 8 | 4.9 | 4 |

There can be no reasonable doubt that the pressure is a direct function of the velocity, since it has been proved by all experiments

The pressures given above are those at the valve. In 1899, Mr. Smith after a number of experiments

concluded that the pressure decreases from the valve toward the free end, as the square root of the distance from the valve. Mr. Seymour '03 made experiments which agreed with those of Smith. Mr. Eide '04, experimenting with the same length of pipe, (300 feet) failed to find any such variation. Joukowsky, with pipe from 1000 to 2000 feet long, found no decrease in pressure until within a short distance from the entrance

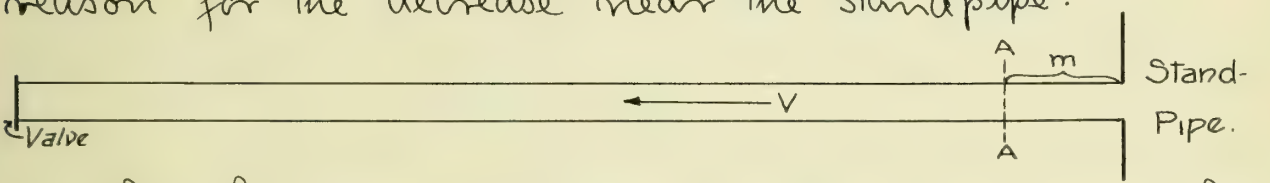
The writer found no decrease in pressure whatever until within 44 feet of the standpipe, and even here the pressure was not decreased when the valve was closed very rapidly. (.02 sec. or less.) The time of closure which affects the pressure is not the time of closing the valve, but is the time of closing the last part of opening. The following table shows the effect of the time of closure on pressure near standpipe.

PRESSURES 44 FT. FROM STAND-PIPE.

| Ref. No | Theoretical Pressure | Actual Pressure | Time of closing Valve — Sec. |
|---|---|---|---|
| 1 | 298 | 210 | .029 |
| 2 | 298 | 260 | .026 |
| 3 | 298 | 260 | .025 |
| 4 | 298 | 260 | .024 |
| 5 | 298 | 290 | .022 |
| 6 | 298 | 300 | .018 |
| 7 | 298 | 320 | .015 |

The last two readings are high because of the inertia of the indicator pencil. However, the table shows in a general way that the time of closure does influence the pressures near the standpipe.

A little consideration of what happens in the pipe after the valve is closed will make clear the reason for the decrease near the standpipe.




When the valve just begins to close, a slight checking of the velocity dV occurs. This decrease in velocity may be considered as a velocity in the opposite direction, and will be represented by +dV (from right to left being negative) This condition will be transmitted through the pipe as a wave having a velocity of 4200 feet per second. After a time t the valve will be completely closed, and the velocity of the water, V, near the valve is completely retarded. This condition will also travel along the pipe with a velocity of 4200 feet per second, and having started t seconds later, will be 4200t feet behind the condition +dV. The velocity of 4200 is only for steel pipe of small size.

When the condition +dV arrives at the stand-pipe, it brings a pressure, as we have seen of 59.7 V (for 2 inch iron pipe) in excess of the static pressure at this point. This immediately starts a flow toward the stand-pipe of -dV. This condition is propagated toward the left with a velocity of 4200 feet per second. When this last condition reaches A-A, (Supposing A-A to be near standpipe) the decrease in velocity coming from the left will have reached some value, +V', and the resultant will be V'−dV. That is to say the decrease in velocity, and hence the pressure will be diminished. As time goes on, +V' increases to +V and -dV to -V, therefore the resultant will at no time be as great as V, and consequently the pressure will never reach a value P. This is on the assumption that the distance m from A-A to the standpipe is such that

$$\frac{2m}{4200} < t$$

To make this still clearer, the writer has translated the following from Joukowsky's monograph, "Stoss in Wasserleitungsröhre"

## Theoretical Determination of the Form of the Diagrams at different Points of the Pipe

This determination was made so as to recognize the conditions which $v$ and $p$ must satisfy, at the valve and open end of the pipe, after the valve has been closed. Just before closure $v$ has a constant value $v_0$ throughout the pipe; and we will assume for simplicity that there is a constant pressure $p_0$ at all points of the pipe before closure; and that it differs only a small amount from the main or standpipe with which the pipe is connected (which is very nearly true when the flow is throttled). If $\tau$ is the time of closing the valve, the velocity $v$ at the valve is decreased from the beginning of closing the valve to the end as a function of the time

$$v = f(t)$$

which function depends on the manner of closing the valve.

If the main, or standpipe to which the pipe is connected has a very large diameter, the pressure at the entrance will always

be constant. That is p will equal $p_0$. For greater simplicity we will consider $P = p - p_0$ instead of the quantity p, and remember that P is equal to zero before closing the valve, and is always zero at the entrance from the main or standpipe.

We will assume that the quantities v and P are made up of the sums

$$v = v_1 + v_2 \qquad P = P_1 + P_2$$

The phase of the conditions $v_1$ and $P_1$ being propagated through the pipe toward the right with a velocity $\lambda$; and the conditions $v_2$ and $P_2$ to the left in a similar manner. For convenience the waves will be referred to as "right" and "left" waves.

We will construct a diagram for the "right" and left waves, as follows

$$cb = \frac{l}{\lambda}$$

which is the time required for a wave to run through the pipe. Taking cb (Fig. 1.) as half the base of the rectangle abde, whose altitude is

$$co = v_0$$

we draw a curve cf, whose ordinates measured

downward from the horizontal line oe determine the velocity of the water past the valve, during closure from

$$v = f(t)$$

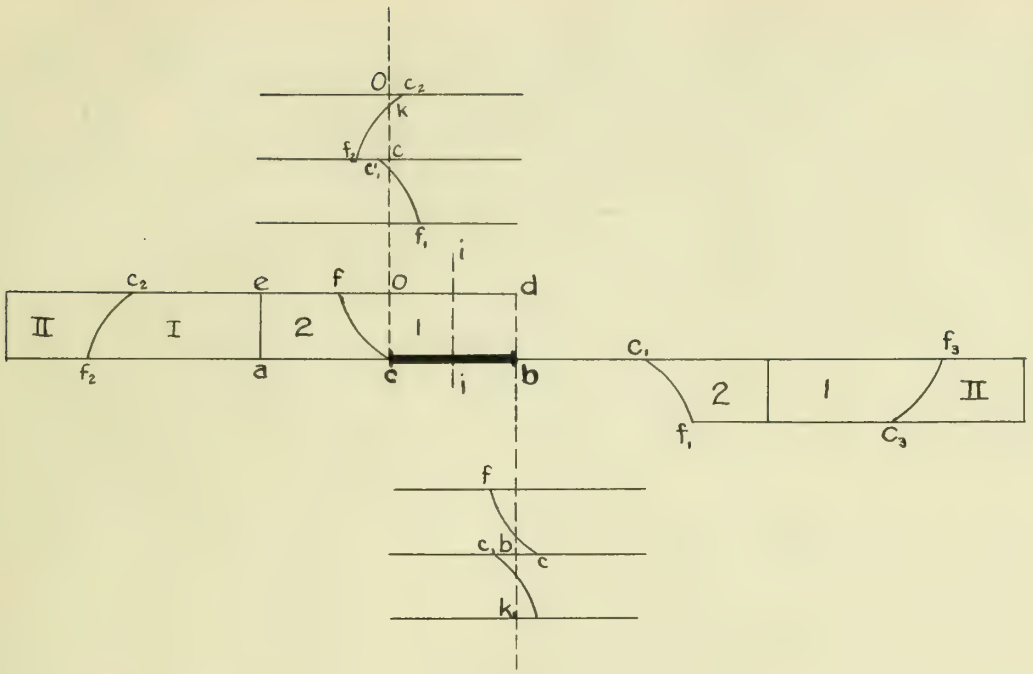
The abscissas t are measured on oe from o to e, so that,

$of = \tau$, the time of closure

The curve cf divides the rectangle into two figures (1) and (2). The figures thus formed are parts of the "right" and "left" waves. The Roman figures I and II show the rectangle abde turned over and moved to the left to complete the wave. Above the horizontal line passing through the pipe cb (Fig. 1.) is the diagram of the right wave, which is put together as shown from (2), (I), (II) and (1.); while below the horizontal line the diagram of the left wave is put together as shown, so that $c_1b = cb$

The value of v in the right wave is given by the vertical distance ed from $bcfc_2f_2$, and the pressure P is given in agreement with the formula

$$P = \frac{v_0 \lambda y}{g}$$


O
c₂
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f₂
c
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i
c₂
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f
O
d
II
I
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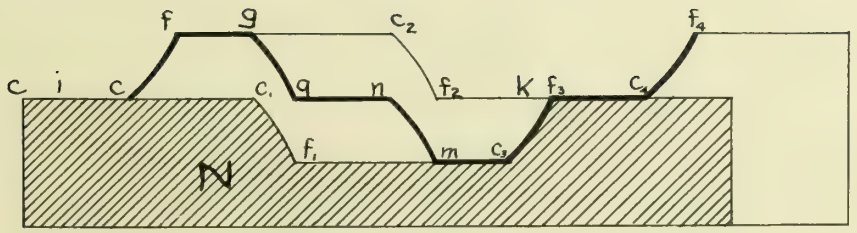
1
f₂
a
c
i
b
c₁
f₃
2
1
II
f₁
c₃
f
c, b
c
k

FIG. 1.


f
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K
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c₄
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m
c₃
N

FIG. 2.

by the distance from $bcfc_2f_2$ to the horizontal line $ab$, multiplied by $\frac{\lambda y}{g}$ or by $\rho\lambda$. The quantity $v$ in the left wave is given as negative values of the distances from the line $cc_1f_1c_3f_3$ to the horizontal line through $ab$, and the pressures are given as negative distances, multiplied by $\rho\lambda$

It is easy to see that the diagrams for the right and left wave satisfy the given conditions at the beginning and end of pipe. Assume that the diagrams move with unit velocity (unit velocity being $\lambda$); that one moves to the right and the other to the left; then we find at the instant of beginning of closure, $v = v_0$ and $P = 0$ throughout the pipe; later after valve has closed somewhat, we get velocities which are represented by the distance of the points on the curve from the horizontal line $ed$, that is, the changes of velocity are according to the law $v = f(t)$. After complete closure of the valve, the velocity $v_0$ and the pressure $P = v_0\rho\lambda$ is transmitted through the pipe. At the moment when the point $c$ of the right wave comes to the open end

of the pipe, the point $c_1$ of the left wave also arrives. From this moment the combination of the positive pressure $\rho\lambda bk$ (See Fig. 1. under point b.) which is brought by the right wave, with the negative pressure $-\rho\lambda bk_1$ which is brought by the left wave, commences. Since $bk = bk_1$, the combination for the value of P makes $P=0$, which will always occur at this point, as the right wave brings the value $P_1 = v_0\rho\lambda$ to the point b, and the left wave the value $-v_0\rho\lambda$.

When the point $c_2$ of the right wave reaches the point O, and the point $c_1$ of the left wave reaches the point c, the combination of the positive velocity $ok$, brought by the right wave with the negative velocity $ck_1$, which is brought by the left wave, begins. (See Fig. 1. above point c.) Since at all times $ok = ck_1$, then $v=0$ at all times at the value. Continuing in this way we will find that all the time after the closing of the value, the pressure at the entrance of the pipe is $P=0$, and at the value $v=0$. We can now construct a diagram for any point of the pipe which has a distance $\xi$ from the value. Let

ci be the distance, then

$$ci = \frac{\xi}{\lambda}$$

the time for the wave to run this distance.

To get the required pressure we must add to every value of P taken from the diagram of the left wave a distance t to the right of i, the quantities P taken from the diagram of the right wave a distance t to the left of i. This may be more easily done by folding the right wave over on the left wave, about i-i as an axis, as is seen in Fig. 2 in which the curves are shown with the same letters as in Fig. 1.

We can readily see that the diagram traced by an indicator will have the shape shown by the heavy line $cfgqnmc_3f_3c_4$. A convenient method for the construction of such diagrams for different points of the pipe is as follows. First make a rectangle N and cut off the part above the curve $cc_1f_1c_3f_3c_4$ of the left wave; then turning the rectangle over, trace on paper $cfc_2f_2c_4f_4$ and place rectangle below it as shown so that

$$cc = 2\frac{\xi}{\lambda}$$

Determining the algebraic sum $P_1 + P_2$ of the various points, the curve

$$cfgqnmc_3f_3c_4$$

may be drawn, and will represent the required indicator diagram

The length of the curve will be

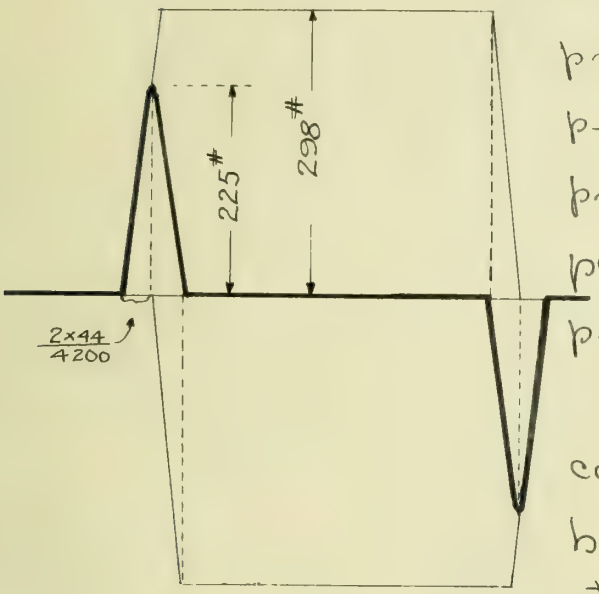
$$cc_4 = 4\frac{l}{\lambda}$$

that is, after four times the time required for wave to run through pipe, the second wave commences. Whereby as is seen in the construction, the diagram will repeat periodically. [Prof. Joukowsky is wrong in this; the diagrams recur, but at diminishing time intervals. See page 70]

As a test of this graphical method we will attempt to produce a diagram similar to one we have taken at 44 feet from the valve. Assume a velocity of 5 feet per second, and a time of closure of .029 second we should get a diagram similar to No. 1 of table on page 36. In making the figure it was assumed that the indicator pencil rose from O to P in a straight line. This is

nearly true for a time of closure less than .03 second.

The diagram resulting is shown by the heavy line. The maximum pressure measures 225 pounds per square inch. The pressure on diagram No. 1 page 36 was 210 pounds per square inch.

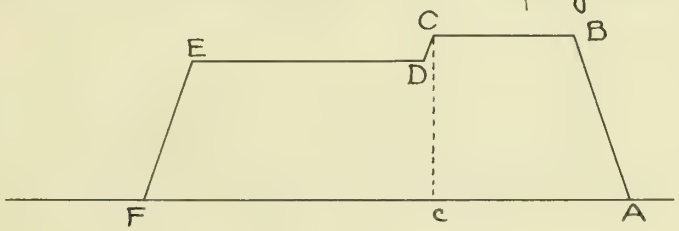



The second wave of course could not go below the sum of the static and atmospheric pressures, and in fact, the diagrams taken by writer have shown no indication of such a wave.

## Determination of the Location of a Leak by Means of the Indicator Diagram

The determination of the place of a leak is often a matter of great importance. When the pipe is laid through a bed of sand, there may be no surface indication whatever of a broken pipe.

On page 68, are shown a few characteristic indicator diagrams of this sort. The general appearance is as shown in figure




The distance Ac is a measure of the time it took the first impulse to travel from valve to leak and back. The distance to the leak can then be found by measuring Ac and substituting in

$$d = \frac{s}{2k}\lambda$$

d being the distance, s the measured distance Ac, k the velocity of the paper to same scale as s, and $\lambda$ the velocity of transmission of the shock through water (about 4200 feet per second in 2-inch iron pipe.)

A number of experiments were made to discover with what accuracy the distance to a leak in the pipe line, could be measured from an indicator diagram taken at the valve. The leak consisted of a 3/4 inch pipe connected into the system, and provided with a valve so that various rates of leakage could be experimented on. Points 72 and 381 feet from the valve were tried.

The following table gives the result of the experiments.

| Ref No | Calculated Dist FEET | Actual Dist FEET |
|---|---|---|
| 1 | 64 | 72 |
| 2 | 70 | 72 |
| 3 | 371 | 381 |
| 4 | 385 | 381 |
| 5 | 375 | 381 |
| 6 | 262 | 265 |
| 7 | 265 | 265 |
| 8 | 116 | 113 |

The last three leaks were from pet cocks, which shows the possibility of measuring the

distance to a very small leak. Nos. 7 and 8 were measured from the same diagram, shown on page 68 The method used in the case of such small leaks was to throttle the flow - and to use a weak spring in the indicator. A 40 pound spring was used in these experiments

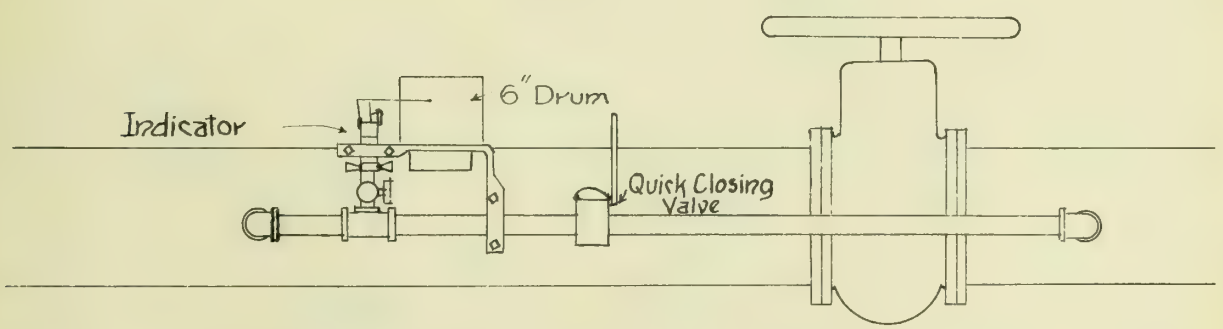
## Suggestions for the Apparatus to be Used in Locating Leaks in a Main.

The work would have to be done at a valve, which would have to be shut off. Since the valves used on mains are very slow-closing it could not be used to produce water hammer. To accomplish this a 1 or 2-inch pipe should be tapped in and led around the big valve. A quick-closing valve in this pipe would produce a sufficient water hammer for our purpose. Some form of valve operated by a heavy spring, released by a trigger would have great advantages. The pressure should be enough to give a card about an inch high, which would be 30 pounds pressure for

a 30 pound indicator spring, and ought to have no effect on the system.

The apparatus must be compact. A brass drum six inches in diameter, run by a spring and regulated by a governor, as is done in phonographs, would be suitable. An ordinary steam indicator could be used and would be preferable to an hydraulic indicator. Hydraulic indicators are made for great pressures.

To hold drum and spring, a base made of 1/4 inch steel plate bent at right angles, split, and twisted to form clamps for fastening to pipe and indicator, could be used. It is of course important that the drum does not change its position relative to the indicator while a diagram is being taken. The following sketch may make the preceding clearer.




The clamps should be used very lightly on the indicator; just enough to take up motion.

The following is a translation made by the writer from Joukowsky's monograph.

## The Determination of the Location of a Leak by Means of the Indicator Diagram.

When the pressure wave due to the rapid closure of a valve, reaches the leak, the pressure $P = \rho h$ falls to $P'$. This creates velocities in the pipe: between the valve and the leak there is a velocity $\frac{P-P'}{h}$ from the valve to the leak, which reduces the velocity from the standpipe to the leak; and in the part between the leak and the open end there is a velocity $\frac{P'}{h}$ towards the standpipe which combining with the velocity in the pipe from the opposite direction, gives a velocity

$$v + w - \frac{P'}{h} = w + \frac{P-P'}{h}$$

towards the leak.

The amount which flowed from the leak before closure was, per second

$$\frac{\pi d^2}{4} w = \mu \frac{\pi d'^2}{4} \sqrt{\frac{2pg}{\gamma}}$$

$d$ and $d'$ being diameters of pipe and leak, $\mu$

the coefficient of discharge from the leak and p the pressure before water-hammer. The phase with the pressure P' and the velocity

$$\frac{P-P'}{h}$$

will be transmitted to the valve, and there cause a negative pressure $P_1$, which will be

$$P_1 = P' - \frac{P-P'}{h}h = 2P' - P = P - 2(P-P')$$

so that

$$2P' = P + P_1$$

and

$$P > P' > P_1$$

Therefore the value of Q in the above formula may be written

$$Q = \frac{\pi d^2}{4}\left(w + \frac{P-P_1}{h}\right)$$

or

$$Q = \mu\frac{\pi d'^2}{4}\sqrt{\frac{2(P'+P)g}{\gamma}} = \mu\frac{\pi d'^2}{4}\sqrt{\frac{2P-(P-P_1)+2P}{\gamma}g}$$

Equating the values of Q we will get an equation from which to determine $P-P_1$ (1)

$$\left[\frac{P-P_1}{h}\right]^2 + \left(\frac{P-P_1}{h}\right)\left\{2\mu\left(\frac{d'}{d}\right)^2\sqrt{\frac{2pg}{\gamma}} + \mu^2 h\left(\frac{d'}{d}\right)^4\frac{g}{\gamma}\right\} - 2\mu^2\left(\frac{d'}{d}\right)^4\frac{pg}{\gamma} = 0$$

When the ratio $\frac{d'}{d}$ is very small and P is large with respect to p, the following approximation may be written

$$P - P_1 = 93.3\left(\frac{d'}{d}\right)^2\sqrt{P} \quad \text{.....(2)}$$

in which $P$ and $P_1$ are expressed in atmospheres.

Assuming that $s$ is the distance from the valve to the leak, and the total length of the pipe is $l$, then $\frac{l}{s} = n$, the number of reflections from the end at valve, before the shock reflected from standpipe reaches the valve.

We will call $P_2, P_3, P_4$ ---- the pressures of the second, third etc. reflections at the valve, and $P''$, $P'''$ --- the second, third ---- pressures at the leak

From the equation $2P' = P + P_1$ we obtain the following set of equations

$$\begin{cases} 2P' = P + P_1 \\ 2P'' = P_1 + P_2 \\ 2P''' = P_2 + P_3 \\ 2P'''' = P_3 + P_4 \end{cases} \quad \text{----------(3)}$$

from which is obtained

$$\begin{cases} P_1 = P - 2(P - P') \\ P_2 = P - 2(P' - P'') \\ P_3 = P_1 - 2(P'' - P''') \\ P_4 = P_2 - 2(P''' - P'''') \end{cases} \quad \text{----------(4)}$$

The pressures are in order of magnitude:

$P, P_1, P_2, \ldots\ldots P', P'', P''', \ldots\ldots$

When the pressure $P_1$ reflected from the valve

reaches the leak, which has a higher pressure $P'$, the pressure at the leak changes to $P''$, so that $P'' < P'$.

From the standpipe a new quantity of water

$$\frac{\pi d^2}{4} \frac{P''-P'}{h}$$

begins to flow toward the leak, and in the part between the valve and the leak a quantity of water

$$\frac{\pi d^2 (P''-P)}{4h}$$

will flow toward the valve, while before a quantity

$$\frac{\pi d^2}{4} \frac{(P-P')}{h}$$

flowed toward leak.

In this manner the quantity $Q_1$ per second, which flows through the leak is determined by the following equation

$$Q_1 = Q + \frac{\pi d^2}{4h} [2(P'-P'') + P_1 - P]$$

using equation (3)

$$Q_1 = Q + \frac{\pi d^2}{4h} (P_1 - P_2)$$

Since $P'' < P'$ it follows that $Q_1 < Q$ so $P_2 > P_1$.

From these inequalities and the formulas (3) and (4) we get

$$P_2 < P \qquad P_2 > P''$$

If after the second reflection from the valve the pressure $P_2$ reaches the leak, then the determinate pressure $P''$ changes to $P'''$ so that

$$P''' > P''$$

In the part of the pipe toward the standpipe or main, a new quantity of water

$$\frac{\pi d^2}{4}\left(\frac{P''' - P''}{h}\right)$$

will flow from the leak, and from the part of the pipe toward the valve, there will be a flow from the leak

$$\frac{\pi d^2}{4}\left(\frac{P_2 - P'''}{h}\right)$$

Before this a quantity

$$\frac{\pi d^2}{4}\left(\frac{P'' - P_1}{h}\right)$$

flowed from the leak

The quantity of water flowing from the leak per second, will increase and become $Q_2$

$$Q_2 = Q_1 + \frac{\pi d^2}{4h}\left[2(P'' - P''') + P_2 - P_1\right]$$

or using (4)

$$Q_2 = Q_1 + \frac{\pi d^2}{4h}\left[P_2 - P_3\right]$$

Since $Q_2 > Q_1$ $\quad P_3 < P_2$

and from formulas (3) and (4) we obtain the following inequalities

$$P_3 > P_1 \text{ and } P_3 < P'''$$

Continuing in this manner we come to the conclusion that

$$P - P_1,\ P_1 - P_2,\ P_3 - P_4, \text{---------}$$

$$P' - P'',\ P'' - P''',\ P''' - P'''', \text{---------}$$

consist of rows of constantly diminishing remainders

The quantity per second flowing from the leak in succeding intervals of time, can, as we have proved, be placed in the following form

$$\begin{cases} Q = \frac{\pi d^2}{4}\left(w + \frac{P - P_1}{h}\right) \\ Q_1 = \frac{\pi d^2}{4}\left(w + \frac{P - P_2}{h}\right) \\ Q_3 = \frac{\pi d^2}{4}\left(w + \frac{P - P_3}{h}\right) \end{cases} \text{----------} (5)$$

These equations show that the velocity of flow from the leak alternately increases and diminishes.

In using (5) the $P - P_i$ factors may be determined by equations similar to (2) and (3)

After n reflections from the valve, the pressure reflected from the end gets to the leak sooner than the pressure reflected from the valve. This wave brings leak the pressure $-P'$ and its corresponding velocity

$$\frac{P'}{h}$$

which is directed to the open end. At the leak a pressure $P^{(n+1)}$ develops, and to the quantity

$Q_{n-1}$ which has been flowing from the leak, we must add

$$\frac{\pi d^2}{4}\left[-\frac{P'}{h}+\frac{P^{(n)}-P'-P^{(n+1)}}{h}+\frac{P^{(n)}-P^{(n+1)}}{h}\right]=\frac{\pi d^2}{4h}\left[2P^{(n)}-2P'-2P^{(n+1)}\right]$$

In this way we will get

$$Q_n=\frac{\pi d^2}{4}\left[w+\frac{P-2P'+2P^{(n)}-P_n-2P^{(n+1)}}{h}\right]=\frac{\pi d^2}{4}\left[w+\frac{-P_1+P_{n-1}-2P^{(n+1)}}{h}\right]$$

This formula shows that $P^{(n+1)}$ is either negative or satisfies the inequality

$$2P^{n+1}<P_{n-1}-P_1$$

The change of pressure at the leak, will be propagated toward the valve, the pressure being $P^{(n+1)}-P^{(n)}$ and the velocity

$$\frac{P^{(n)}-P^{(n+1)}}{h}$$

which is directed towards the leak.

This wave brings a negative pressure to the valve, which is

$$P_{n+1}=2P^{(n+1)}-2P^{(n)}+P_n$$

Because of the preceding inequalities this pressure must satisfy

$$P_{n+1}<P_{n-1}-P_1-2P^{(n)}+P_n$$

or by (3)

$$P_{n+1}<-P_1$$

In this way a negative pressure is developed at the valve, and the diagram will show a sharp fall in pressure at this point. The total

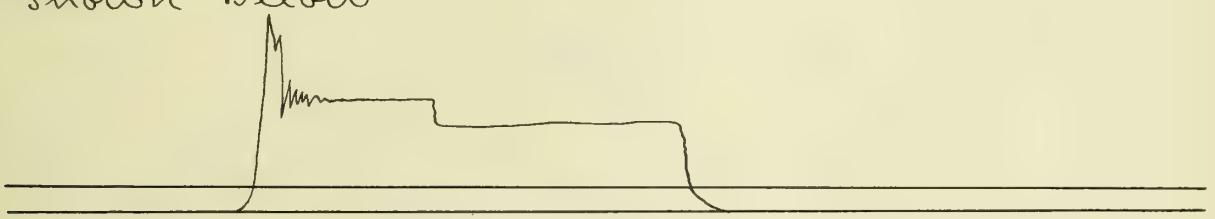
length of the pressure wave on the diagram taken at the valve will be proportional to

$$\frac{2l}{\lambda}$$

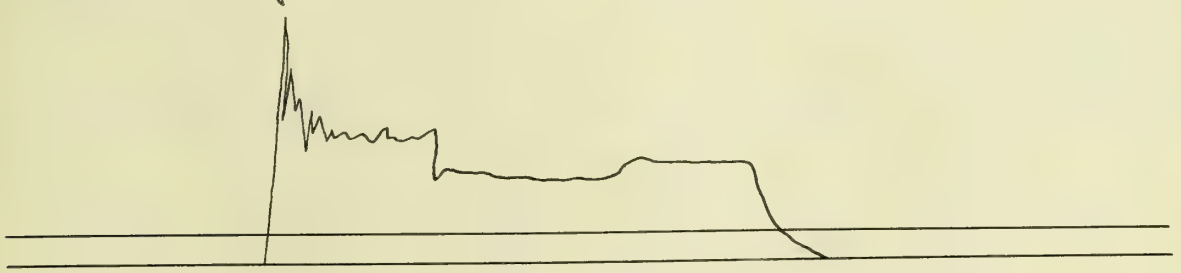
The pressure will consist of $n+1$ degrees, of which the first $n$ have a length

$$\frac{2s}{\lambda}$$

These degrees have the heights $P, P_1, \ldots\ldots$ whose differences $P-P_1, P_1-P_2, \ldots$ become always less and are so shown on the diagram. If $n=1$, that is if the leak is nearer the open end than the valve, the diagram will consist of two degrees, and will look as shown below



The next figure shows a diagram of three degrees ($n=2$)



. . . . .

## Oscillations Following Rapid Closing of a Valve at the End of a Long Pipe

When the valve at the end of a pipe is closed, a series of pulsations follow. The pressure of each succeeding pulsation and the time between them become less and less. The diagram on page 70 shows this clearly.

For the 727 feet of 2-inch pipe used by the writer the following empirical formulas fit all the cases tested

$$P_n = .836\, P_{n-1}$$

and

$$T_{n+1} = .634 \frac{P_{n-1}}{p+\alpha} + .519$$

in which $P_n$ and $P_{n-1}$ are the $n$ th and $(n-1)$th pressure waves, $T_{n-1}$ the time between them, $p$ the pressure on entrance to standpipe, and $\alpha$ the atmospheric pressure. $P_n$ etc being expressed in pounds per square inch, $p$ and $\alpha$ in feet of head.

The column acts as a spring, and when it is much compressed it takes it longer to expand and be forced back than when only slightly compressed

## Movement of Water after the Valve is Closed

The following is an extract from Seymour's thesis:

"It is interesting to consider what happens after the valve is closed. Let $\lambda$ equal the amount of compression for a length x, then $\lambda = \frac{Sx}{E}$, where S equals the unit-pressure, and is equal to P, E the modulus of elasticity of water. When S = 100 pounds and E = 300 000, $\lambda = \frac{100x}{300\,000}$ = the compression in a column x feet long. For a column 20 feet long $\lambda = \frac{100 \times 12 \times 20}{300\,000} = .0816$ inches, considering that the pressure is uniform for this distance. Taking the average pressure for each 20 feet of pipe from diagrams on Plates 1 and 2, the approximate amount of compression for each 20 feet is found. Adding these amounts, beginning at the valve, the sum equals the total compression of the column, or the movement of the water at the free end after the valve is closed and up to the time of greatest compression, when the water comes to a standstill. This

total movement divided by the velocity of the water in feet per second will give the time in which the water continues to flow into the pipe after valve is closed, if we disregard the time taken to generate the pressure at any point, which the diagrams show to be very small. The times that the water flows after the valve is closed were calculated as above. We should expect it to be the same as the time it takes the impulse to travel from the valve to the free end, but it is shown to be less. No explanation can be given here."

The explanation of this lies in the following. Mr. Seymour was in error in assuming that the pressure in general varies as the square root of the distance from free end. That his experiments seemed to indicate this to be true is conclusive proof that the effective time of closure was rather large. The writer in experiments with the same valve finds the effective time of closure may range from .015 to .030 seconds, depending somewhat on the velocity of the water, the quicker closure

being with high velocities. This is because with small velocities, a noticeable retardation is produced by a smaller motion of the valve, than with high velocities

The compression near the valve begins some time before the valve is entirely closed. During this time water is flowing through the valve, and this quantity must be taken account of. The quantity flowing will be approximately half of the full flow for the same time, since the velocity of the water changes from V to 0 at a nearly constant rate.

Another error is in not taking into account the effect of the stretching of the pipe. The increase in volume of the pipe due to this cause will be

$$\pi/4\,(D^2-D_0^2)\,12l = 3\pi l(D^2-D_0^2)$$

or since D is nearly equal to $D_0$, $D+D_0=2D_0$

$$3\pi l(D-D_0)(D+D_0) = 6lD_0(\pi D-\pi D_0)$$

but $\pi D-\pi D_0$ is the stretch in the pipe, which is (see page 25.) $\dfrac{\pi D_0^2 P}{2tE'}$

E' is the elasticity of the material in the pipe

hence $6lD_0(\pi D-\pi D_0) = 6lD_0\dfrac{\pi D_0^2 P}{2tE'}$

or finally increase in volume due to stretching

the pipe is

$$\frac{3\pi D_o^3 l P}{t E'} \text{ cubic inches}$$

The water will be compressed

$$\frac{P}{E} l \tfrac{1}{4}\pi D_o^2 = \frac{\pi D_o^2 P l}{4E}$$

The ratio of the increase in volume of the pipe due to stretching, to the amount of compression of water, will be

$$\frac{3\pi D_o^3 l P}{t E'} \cdot \frac{4E}{\pi D_o^2 P l} = \frac{12 D_o}{t} \frac{E}{E'}$$

which for 2-inch pipe is 15.4%

The following table shows Mr. Seymour's results

| 300 feet of pipe | | | | 180 feet of pipe | | | | |
|---|---|---|---|---|---|---|---|---|
| V = 7 | V = 5.4 | V = 22 | Theo. Time | V = 9.2 | V = 7.7 | V = 5.6 | V = 2.2 | Theo. Time |
| .0486 | .0457 | .0365 | .063 | .0262 | .0260 | .0253 | .0178 | .038 |

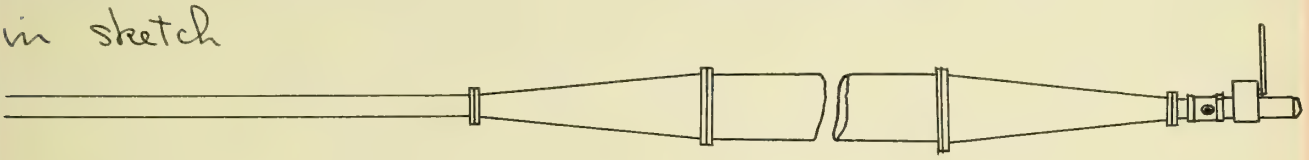
The error in the 300 foot pipe is considerably greater than in the 180 foot pipe. This is because with a closure in .02 seconds, the pressure should be the same from the valve to a point about 42 feet from the end, and should then fall off to zero pressure at the end. The error, therefore in assuming that

the pressures vary as the square root of the distance, would be greater for 300 feet of pipe than for 180 feet of pipe.

When the values in the table are corrected for the time of closure (by adding half the time), for the neglecting of the stretching of the pipe, and the error made in assuming pressures follow the parabola law, the times will agree.

# The Effect of Placing a Pipe of Larger Diameter in the Pipe Line.

Experiments were made with ten feet of 6-inch pipe in the line, and near the valve. A gradual increase and decrease in section was provided, as is shown in sketch

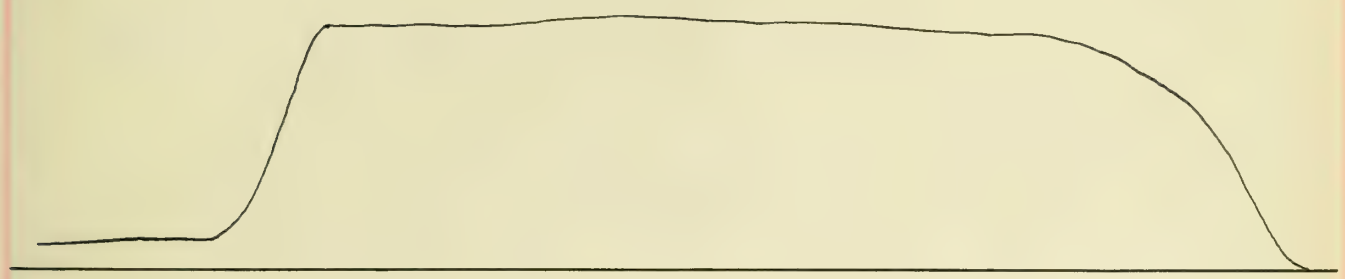


The diagram taken at the valve in this case, has a very gradual rise in pressure at the start. The large pipe gives the effect of a very gradual closure. This is because when valve is closed, the water in the six inch pipe is stopped, but its velocity being only one-ninth that in the two inch pipe, the compression per unit volume will be only one-ninth. When the pressure wave gets into the two inch pipe there will be a flow into the larger pipe until equilibrium is attained.

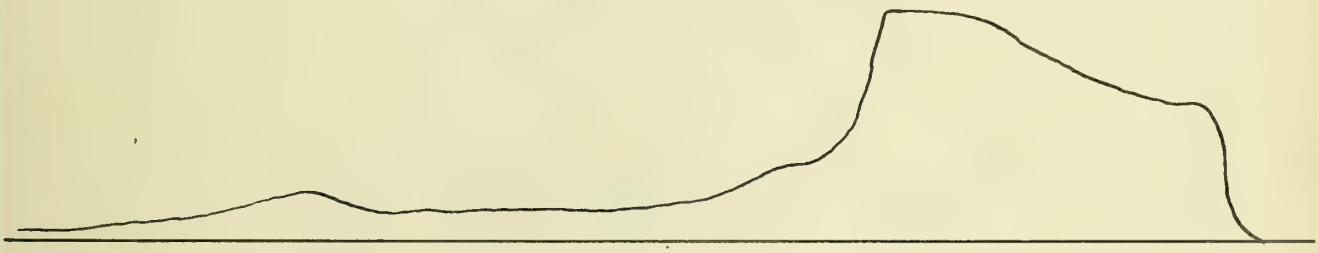
The appearance of the diagram is shown on page 67

# DIAGRAMS WITH SIX INCH PIPE IN THE LINE

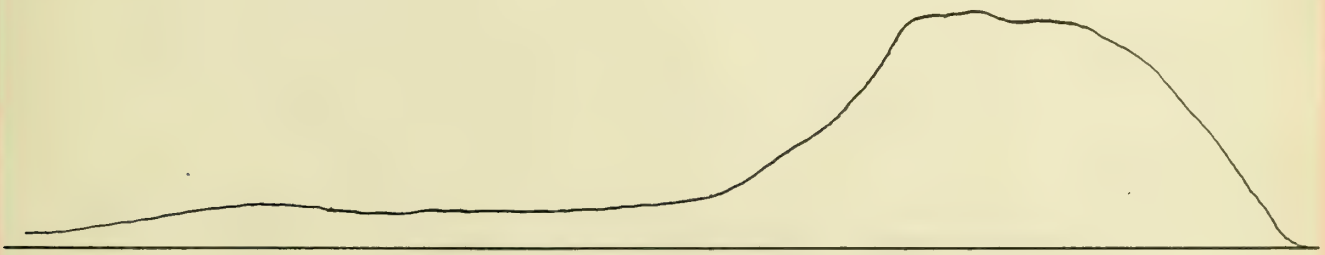
See page



At valve

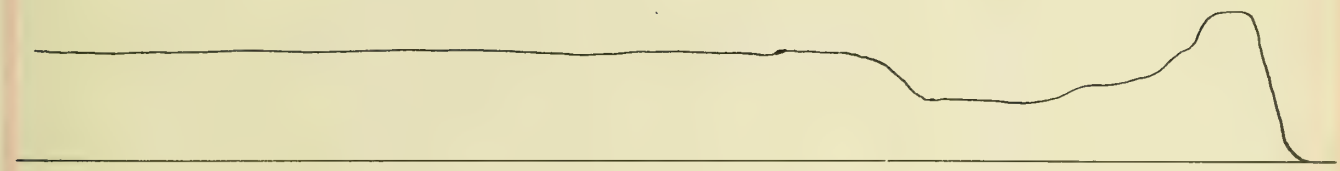


267′ from standpipe

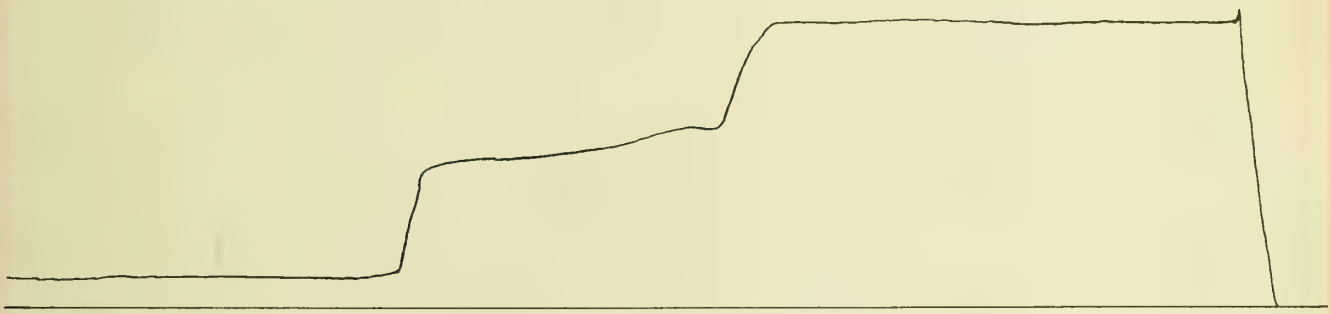


267′ from standpipe

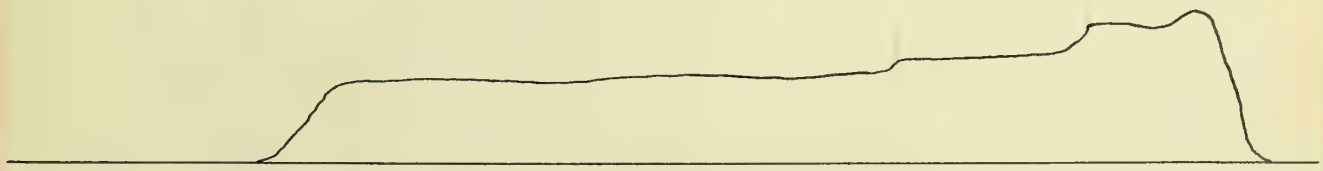
# DIAGRAMS SHOWING LEAKS.



Leak 72′ from valve



Leak 381′ from valve

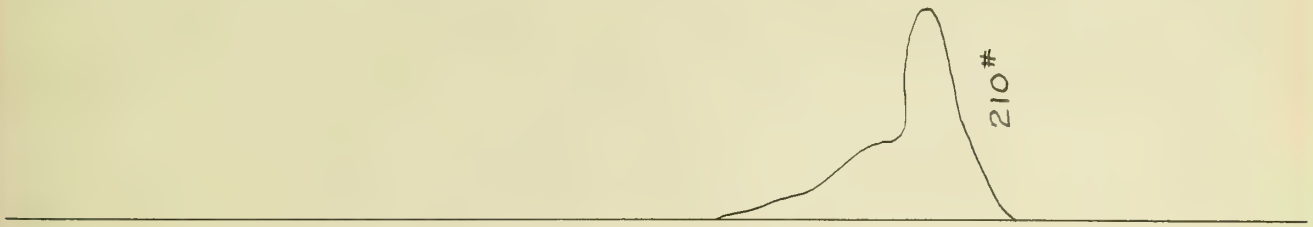


Petcocks 113′ and 265′ from valve open

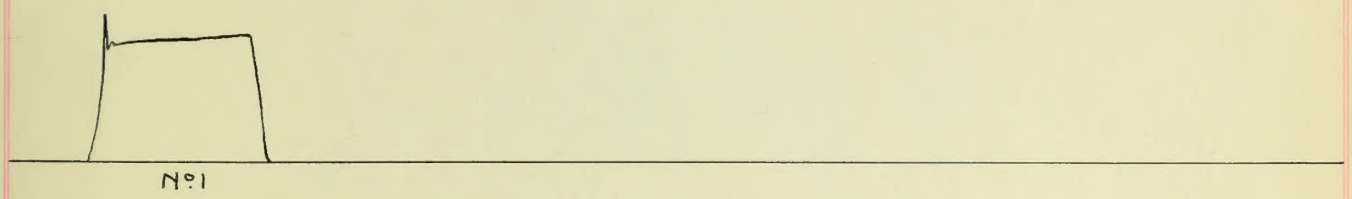
# DIAGRAMS 44 FT. FROM STANDPIPE

V = 5 FEET PER SEC

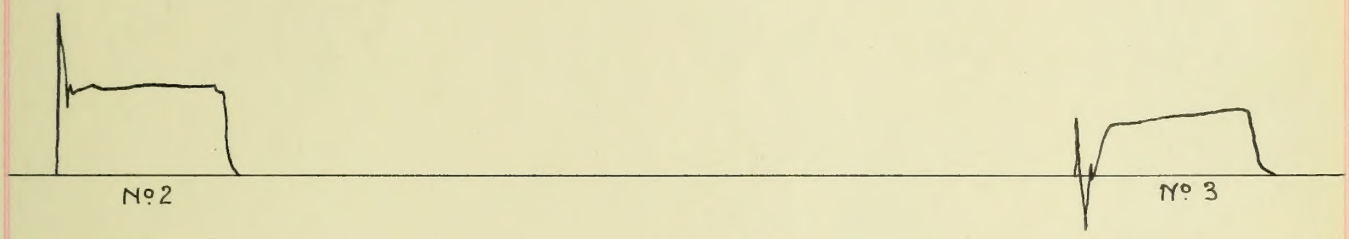
Showing effect of time of closure on pressure.



# DIAGRAM SHOWING SUCCESSION OF WAVES

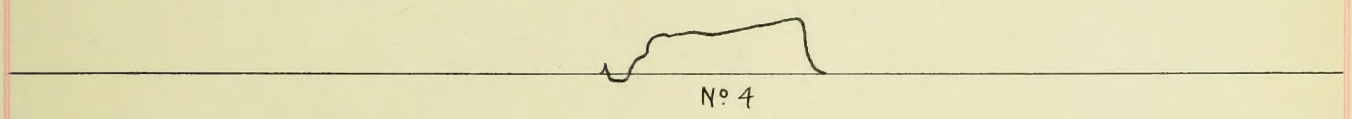


Nº 1



Nº 2

Nº 3



Nº 4

Nº 5

Nº 6.

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